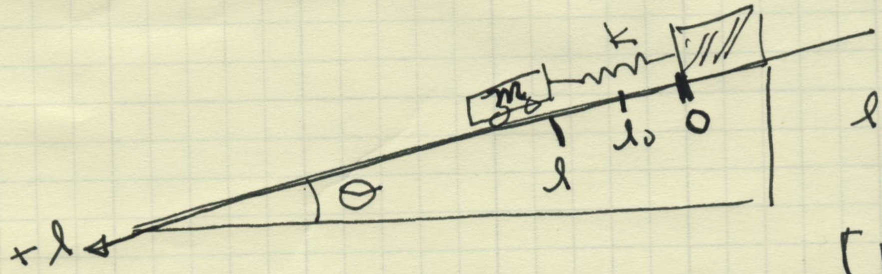
SPRING-RAMP-CART ①/3



l_0: unstretched length of spring

$[F_{sp} = -k(l - l_0)]$

m : mass of cart.

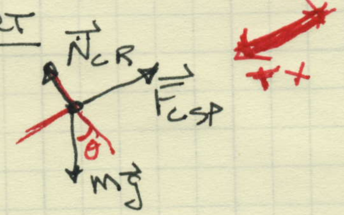
(a) cart is in Equilibrium @ position l.

$l > l_0$; so spring is stretched [elongated] $\Delta l = (l - l_0)$.

Q $\boxed{\text{find } l : l_0, m, k, \theta \text{ \& } g.}$

S. System : 1 Body The CART

I. Interactions : FBD



M. Model : NII : $\Sigma \vec{F} = m\vec{a}$.

since Equilibrium Means $a = 0$, $\Sigma \vec{F} = \vec{F}_{net} = 0$.

Use FBD and tilted coord. System drawn

MODEL

A straight-forward pt-Body Application of Newton's 2nd

$\Sigma F_x = mg\sin\theta - F_{spring} = 0$

No need for Rest of $\Sigma \vec{F}$.

so $mg\sin\theta = +k\Delta l$

$mg\sin\theta = k(l - l_0)$

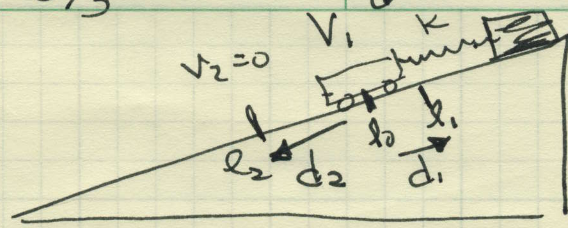
$\frac{mg\sin\theta}{k} = (l - l_0) = \Delta l$

$$\boxed{l_0 + \frac{mg\sin\theta}{k} = l}$$

is you can't read it.

$$\boxed{l_0 + \frac{mg\sin\theta}{k} = l}$$

(b)



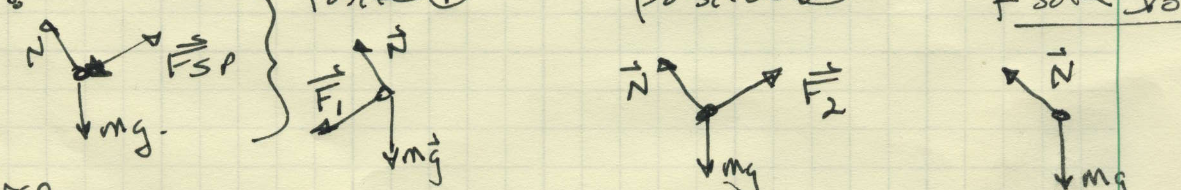
position ② @ lowest position
card displaced d_2, stretch.
$V_2=0$, stops at max displacement d_2

position ①
cart displaced d_1 + compressed
released from rest $V_1=0$

Q: find V @ l_0 as it 1st passes thru equil position on way from $l_1 \to l_0 \to l_2$.
$\underbrace{l_1 \to l_0}_{d_1}$ $\underbrace{l_0 \to l_2}_{d_2}$

System: 1 Body
CART.

II Interactions.
FBD



spring Force
$F = -k\Delta l$
grav. $F_g = mg$
$\vec{N}$

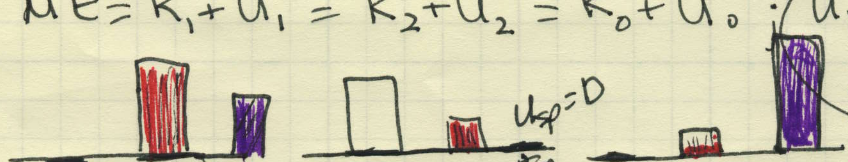
b/c $\Delta l = 0$
so $F_{sp} = 0$.

Model; Energy conservation: No friction $\Rightarrow$ all forces are conservative.

so $ME = K_1 + U_1 = K_2 + U_2 = K_0 + U_0$; $U_1 = U_{g1} + U_{sp1}$ etc...

Both Grav + Spring PE

Bar graph



at 1 K_1 U_{g1} U_{s1} K_0 U_{g0} U_{s0} $= K_2$ U_{g2} U_{s2}

$$\left(0 + mgy_1 + \tfrac{1}{2}kd_1^2\right) = \tfrac{1}{2}mV_0^2 + mgy_0 = 0 + mgy_2 + \tfrac{1}{2}kd_2^2$$

Notice $U_g = mgy$; y_1 decreases as it moves down the ramp.
$U_{sp} = \frac{1}{2}k(\Delta l)^2$; at equil or $l = l_0$, $U_{sp} = 0$; AND d_2 must be $> d_1$

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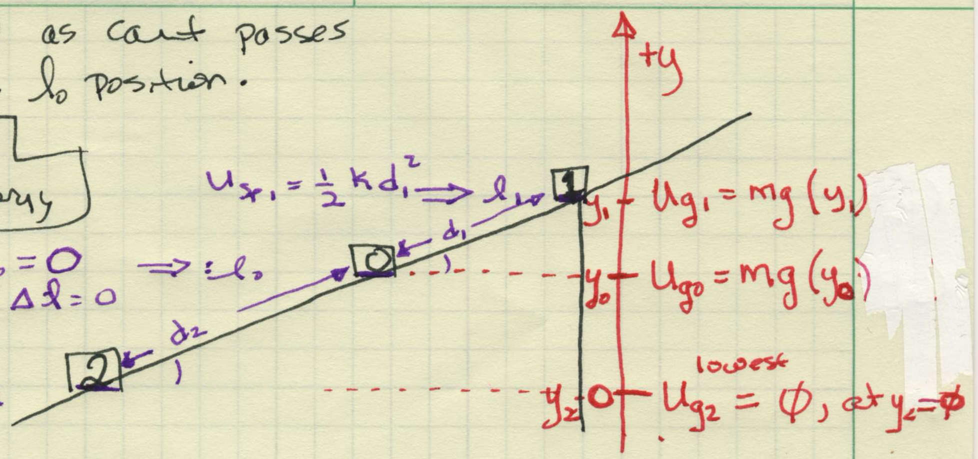
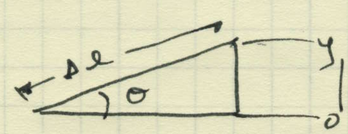
(b) find $\underbrace{V_0}$: speed as cart passes thru l_0 position.

$\boxed{\text{Model: Energy cons. to find Kinetic Energy}}$



$\frac{1}{2}kd_2^2 = U_{sp2}$ | lowest. $\Rightarrow l_2$ position on Ramp.

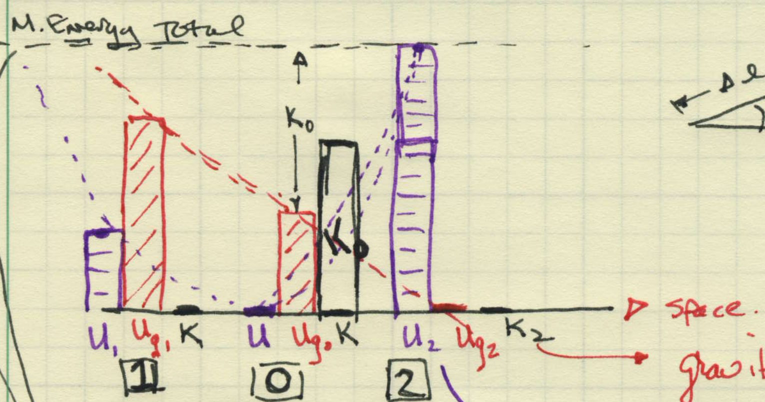
from geometry of Ramp.



$|\Delta y = \Delta l \sin\theta$

$y_1 - y_0 = d_1 \sin\theta$

$y_0 - y_2 = d_2 \sin\theta$



gravitational U_g goes down linearly along ramp.

Spring U_{sp} is quadratic about $l_0 = y_0$ position.

$$E_{TOT@1} = E_{TOT@0} = E_{TOT@2}$$

$$(U_{sp1} + U_{g1} + \cancel{K_1}^{0}) = (\cancel{U_{l0}}^{0} + U_{g0} + K_0) = U_{l2} + \cancel{U_{g2}}^{0} + \cancel{K_2}^{0}$$

$$\frac{1}{2}kd_1^2 + mgy_1 = mgy_0 + \frac{1}{2}mV_0^2 = \frac{1}{2}kd_2^2$$

$$\frac{1}{2}kd_1^2 + mg(y_1 - y_0) = \frac{1}{2}mV_0^2$$

$$\sqrt{\frac{k}{m}d_1^2 + 2gd_1\sin\theta} = V_0$$

here's how fast it goes on the 1st pass thru the spring's Equil. Positn.

(c) if friction $f = \mu N = \mu mg\cos\theta$
Then on first pass from top to l_0,
friction does $W = \vec{f}\cdot\vec{\Delta l} = -(\mu mg\cos\theta)d_1$ on block; reducing the KE at that point by this much energy. So

$$\left(\frac{1}{2}kd_1^2 + mg(d_1\sin\theta) - \mu mg\cos\theta\, d_1\right) = \frac{1}{2}mV_0^2 \Rightarrow V_0 = \sqrt{\frac{k}{m}d_1^2 + 2gd_1\sin\theta - 2\mu gd_1\cos\theta}$$

$$V_0 = \sqrt{\frac{k}{m}d_1^2 + 2gd_1(\sin\theta - \mu\cos\theta)}$$