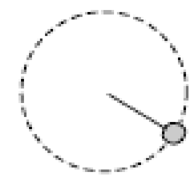
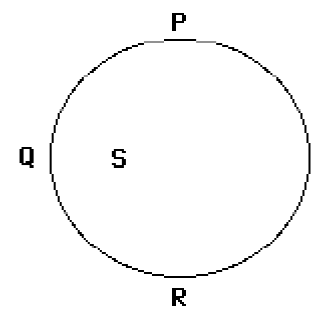
**PSET #4: Circular Motion** skj November 3, 2010

**Problem 1. Short exercise to find the direction of acceleration.**

a) A car moves at constant speed on a flat road when it suddenly encounters a circular hump in the road. What is the direction of the acceleration of the car when it is at the top of the hump? Show your work.



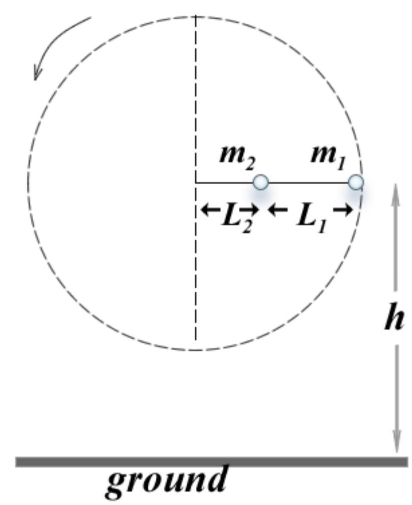
b) A ball tied to a string rotates in a circle counter-clockwise on a vertical plane. The speed of the ball is found to be a maximum at the bottom and minimum at the top. What is the direction of the acceleration vector when the ball is at the 4-o'clock position? Show your work.




c) An object moves counter-clockwise along the circular path shown in the figure. Is it possible to have a situation where an object moves counter-clockwise along a circular path and its acceleration vector continuously points toward point ***S*** in the figure.

Explain the reasoning of your answer. If you answer yes, also give a real life example that can be modeled with the situation described above.

**Problem 2: Two beads and a two strings.**




Two small beads of masses m1 and m2 are connected to each other by a string of length L1. The other end of bead 2 is connected to another string of length L2. A little boy holds the free end of the string (L2) at a height h from the ground and rotates the beads counter-clockwise in a vertical circle as shown in the figure. (the figure does not show the boy).

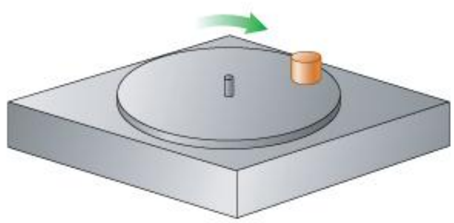
When the beads are at the 3 o'clock position, the string between the two beads (L1) breaks. The bead that flies off returns to the ground in a time interval $\Delta t_1$ after the string broke. The goal is to determine the tension in each of the two strings just before the string (L1) broke.

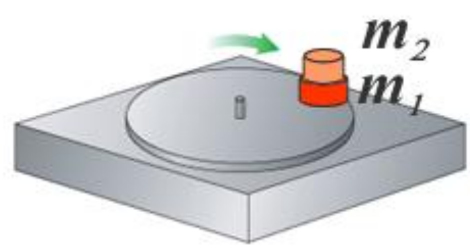
a) Break the motion into stages or distinct time intervals ($\Delta t$'s ) that simplify the descriptions of motion and the conditions that apply during each time interval. For each stage, indicate the system (ie which object or objects) you are considering. For each stage describe how you will model the object(s), draw the FBD to identify all the relevant interactions between objects. For each stage make a representation of the motion : either represent the motion graphically, or by drawing a motion diagram or by sketching the path an annotating it. For each stage specifically state the physical principle(s) you will apply to solve each sub-problem. For each stage state explicitly what you will be solving for in the analysis of that particular stage of the process.

b) Include relevant representations for each stage /sub-problem.

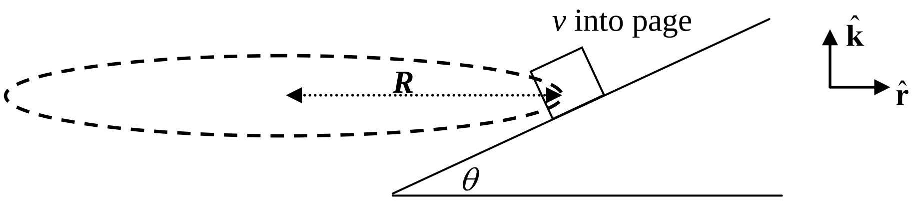
**Problem 3.**

Two metal cylinders of mass $m_1$ and $m_2$ are placed one at the top of the other on a circular turntable that is rotating at a constant speed as illustrated in the diagram. The coefficient of static friction between the lower cylinder and the turntable is $\mu_{s1}$ and the coefficient of static friction between the cylinders' surfaces is $\mu_{s2} > \mu_{s1}$. The cylinders remain at rest with respect to the turntable and are at a distance $R$ from its center. If the angular speed of the turntable increases, at what maximum angular speed $\omega_{max}$ will the cylinders start to move?
(first consider the simpler problem of one mass on the turntable, with $\mu_{s1}$). ***Then work on the more complicated one of m1 wtih m2 sitting on top.***





**Problem 4: Banked turn**




A car of mass m is goes around a circular turn of radius $R$ with constant speed $v$. The turn is banked at an angle $\theta$ with respect to the ground. The coefficient of static friction between the wheels and the road is $\mu_s$. The car's tires do not slip.
*To answer the qualitative parts c) & e), you can construct a demo of this situation. Use a large steel bowl (a mixing bowl) that you can use as the banked track. Use a small ball such as a marble to represent the car. Give the ball an initial speed so that it moves in a circle on the walls of the bowl. Try to construct situations c) and e). The picture above is a vertical cross section of the bowl.*

a) What is the direction of acceleration of the car? Choose a coordinate system so that the one axis points in the direction of acceleration.
b) Draw a free-body diagram for the car.

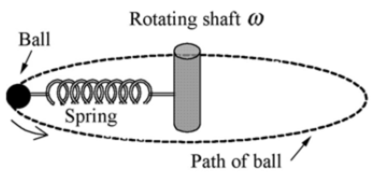
c) Under what conditions will the car **start** slipping down the embanked turn?
d) **Derive** an expression for **the minimum velocity** necessary to keep the car moving in a circle without slipping ***down*** the embanked turn.

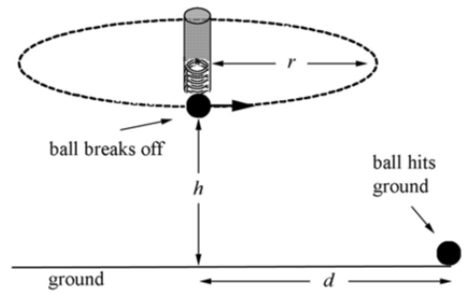
e) Under what conditions will the car start slipping down the embanked turn?
f) **Derive** an expression for the maximum velocity necessary to keep the car moving in a circle without slipping ***up*** the embanked turn.

g) Do limiting case checks to for your answers in d) & f). Especially look at the following two cases – what should happen when there is no friction and what should happen if the angle was zero.
Express your answers in terms of the given quantities and the gravitational constant $g$.

## Problem 5. Synthesis

A ball of mass $m$ is connected by a spring to shaft that is rotating with constant angular velocity $\omega$. A student looking down on the apparatus sees the ball moving counterclockwise in a circular path of radius $r$. When the spring is unstretched, the distance from the mass to the axis of the shaft is $r_0$. The orbital plane of the ball is a height $h$ above the ground. Suddenly the ball breaks loose from the spring, flies through the air, and hits the ground an unknown horizontal distance $d$ from the point the ball breaks free from the spring. Let $g$ be the magnitude of the acceleration due to gravity. You may ignore air resistance and the size of the ball. Express your answers to the questions below in terms of the given quantities $m$, $\omega$, $r$, $h$, $r_0$ and $g$ as needed.



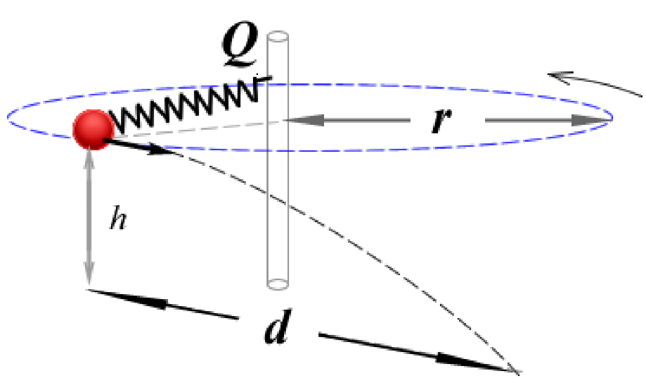

a) Draw a force diagram on the ball just before the ball detaches from the spring. Assume the effects of gravity are negligible.

b) What is the magnitude of the spring constant $k$? Show all your work. Answers without work will not receive credit.

c) What is the magnitude of the velocity of the ball when it breaks free from the spring?

d) How long does it take for the ball to hit the ground? Show all your work. Answers without work will not receive credit.

e) Find an expression for the horizontal distance $d$ the ball traveled from the point the ball breaks free from the spring until it hits the ground. Show all your work. Answers without work will not receive credit.




$\Delta\theta/\Delta t = \omega =$ angular velocity. This is constant, so this means it takes the same amount of time to go around $2\pi$ radians. If the period is T, then omega = $\omega = 2\pi$ radians/T.

**Problem 6. A rotating cone (**a really funky fun problem with no numbers and lots of visualization challenges.)

A small block with mass m is placed inside an inverted cone that is rotating about a vertical axis such that the time for one revolution of the cone is T as shown in the figure. The walls of the cone make an angle β with the vertical. The coefficient of static friction between the block and the cone is $\mu_s$. The goal of the problem is to find the magnitude of the force of friction $F_{fr}$ between the block and the cone's wall if the block is to remain at a constant height *h* above the apex of the cone

**Part A.**

Without solving the complete problem and using limiting cases rule out the incorrect answers from the list below. In order to get full credit you must explain why you rule out or accept any of the expressions listed below. Clearly indicate:

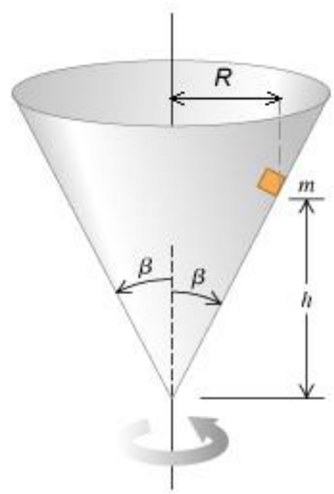
i) the variable(s) you are considering,
ii) which is the numerical limit that you assign to that variable(s), and
iii) what is the expected physical limit case.

(1) $F_{fr} = m|\frac{v^2}{R} sin\beta + g\cos\beta|$

(2) $F_{fr} = m|\frac{v^2}{R} cos\beta + g\sin\beta|$

(3) $F_{fr} = m|\frac{v^2}{R} sin\beta - g\cos\beta|$

(4) $F_{fr} = m|\frac{v^2}{R} cos\beta - g\sin\beta|$




**Part B**

Describe the plan that you would follow to obtain $F_{fr}$ (what, why and how).

Include in your description the system you will use, the objects inside and outside the system, the interactions between the objects and the processes. List all the assumptions about the objects and the interactions and show all relevant representations. Clearly state **what** physical principles you will use, and **how** you will use them. In the **how** part, include the equations (based on the principles) for this specific situation.

**Note:** You need to set up your equations till the point where you can algebraically solve for $F_{fr}$. But you do not need to algebraically solve the equations to get tension, since you already know the answer from part a).