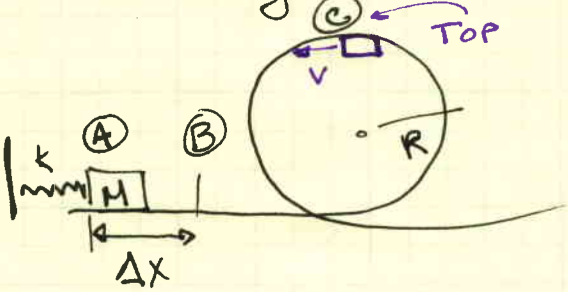
Spring launched Loop-the-Loop With Human Constraints ①/2

$m = 500\,kg$. $V_i = 0$ Shot by spring k up Loop.

$R = 10\,m$.



Launch

2 CONDITIONS

① during Launch $a \le 10g$.

② Top: F by Track on CART.

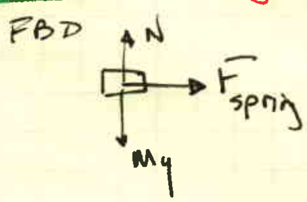
$N = F > \frac{Mg}{2}$ (Force by Track = Normal force)

System during Launch

~~Launch~~: 1 body m

Interactions during Launch.

FBD



Model: NII Law

+ Hooke's Law

$F_{spring} = F_{net} = ma \le m(10g)$.

so $\frac{1}{2}k\Delta x^2 \le 10mg$

$\Delta x = x_0$; compression of Spring.

System at Top

1 Body.

Interactions

F_{hooke} + $F_g = mg$

Model

* Energy conservation:

ME is const

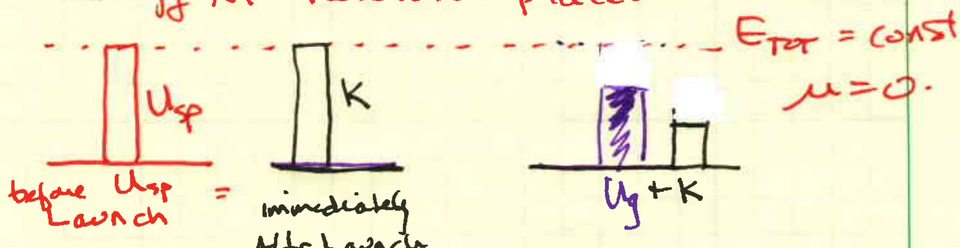
No Non-Cons. Forces

* NII - Circ. Motion

$N \downarrow \downarrow mg$

$F_{net} = N + mg = \frac{mV_c^2}{R}$

Energy AT Relevant places.



Ⓐ Ⓑ Ⓒ

$E_A = E_B = E_C$

$\frac{1}{2}kx_0^2 = \frac{1}{2}mV_B^2 = mg(2R) + \frac{1}{2}mV_c^2$

The KE needed at the top so it doesn't fall off is Related to the Net force on "m" at Top ... and constrained! by ②

Spring Launched loop-the-loop With human constraints

At the Top: Model as pt particle so,

$$F_{net} = N + mg = \frac{mV_c^2}{R}$$

constraint at Top : $N \geq \frac{mg}{2}$

so $\frac{3}{2}mg > \frac{mV_c^2}{R}$

$$\boxed{\frac{3mgR}{2} > mV_c^2.} \rightarrow \text{so} \quad \frac{3mgR}{4} > K_e = \frac{1}{2}mV_c^2$$

Also the constraint that $F_{net\ Launch} < \frac{mg}{2}$

requires $\boxed{Kx_0 < \frac{mg}{2}} \Rightarrow x_0 < \frac{mg}{2K}$

from Energy:

$$\frac{1}{2}Kx_0^2 = 2mgR + \frac{1}{2}mV_c^2$$

so $\frac{1}{2}Kx_0^2 - \frac{1}{2}mV_c^2 = 2mgR$

if E cons.

+ x_0 fits conditions.

the question is ....

given K sufficiently sized;

what x_0 satisfies both

constraints ?

$$\frac{1}{2}Kx_0^2 - \frac{3mgR}{2} < 2mgR$$

Duh!

so $\frac{1}{2}Kx_0^2 < 2mgR + \frac{3mgR}{4}$)

$$\frac{1}{2}Kx_0^2 < \frac{8mgR}{4} + \frac{3mgR}{4} = \frac{11\,mgR}{2}$$

so $x_0^2 < \frac{11\,mgR}{K} \rightarrow \boxed{x_0 = +\sqrt{\frac{11\,mgR}{K}}}$

OR $\boxed{Kx_0^2 < 11\,mgR}$

DESIGN CRITERIA

$Kx_0^2 < 11\,mgR$

$Kx_0^2 < 5.40\times10^5$ Joules